Practice 2

Topic: ***Research ACS on controllability by R. Kallman and E. Gilbert's criteria***

Example. The dynamic system is described in state-space by the system of the equations:

$$\begin{cases} \dot{x} = Ax + Bu \\ y = Cx \end{cases}, \qquad (*)$$

where $A = \begin{vmatrix} -4 & 5 \\ 1 & 0 \end{vmatrix}$, $B = \begin{vmatrix} -5 \\ 1 \end{vmatrix}$, $C = \begin{vmatrix} 1 & -1 \end{vmatrix}$.

The description of a system will be written down in state-space in a matrix form as follows:

$$\begin{vmatrix} \dot{x}_1 \\ \dot{x}_2 \end{vmatrix} = \begin{vmatrix} -4 & 5 \\ 1 & 0 \end{vmatrix} \cdot \begin{vmatrix} x_1 \\ x_2 \end{vmatrix} + \begin{vmatrix} -5 \\ 1 \end{vmatrix} \cdot u .$$

Also we will write the equation in matrix form for an output variable:

$$y = (1, -1) \begin{vmatrix} x_1 \\ x_2 \end{vmatrix} .$$

Check the researched dynamic system for controllability by R. Kallman and E. Gilbert's criteria.

Algorithm and solution

1. We find own numbers of a matrix A.

We write the characteristic equation for a system as follows:

$$\det(A - \lambda I) = 0 .$$

We obtain own numbers of the matrix A:

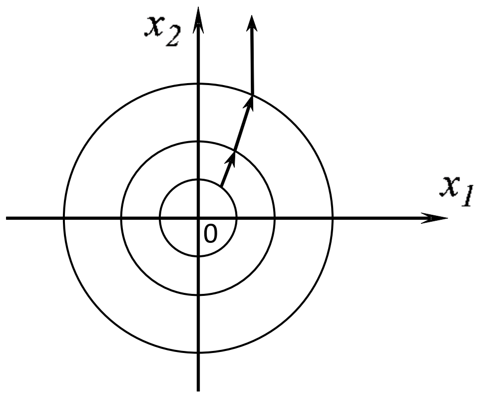
$$\det \begin{vmatrix} (-4-\lambda) & 5 \\ 1 & (-\lambda) \end{vmatrix} = 0;$$

$$(4+\lambda)\lambda - 5 = \lambda^2 + 4\lambda - 5 = 0$$

$$\lambda_1 = -5; \quad \lambda_2 = 1.$$

Hence the movement of the researched dynamic system is unstable across Lyapunov as a real part of the second root is positive, i.e. $\lambda_2 > 0$.

Geometrical interpretation:



I. Check on controllability by R. Kallman's criterion

Algorithm and solution

1. The dynamic system is described in state-space by the system of the equations (*), where the matrixes:

$$A = \begin{vmatrix} -4 & 5 \\ 1 & 0 \end{vmatrix}, \quad B = \begin{vmatrix} -5 \\ 1 \end{vmatrix}.$$

It's necessary to construct a block matrix of controllability for check of property by R. Kallman's criterion.

Matrix of A(2*2), therefore, the order of a system is equal 2, i.e. $n = 2$.

The block matrix of controllability will write down for this system as follows:

$$K_u = (B, AB).$$

2. We define a rank of a block matrix of controllability:

$$AB = \begin{vmatrix} -4 & 5 \\ 1 & 0 \end{vmatrix} \begin{vmatrix} -5 \\ 1 \end{vmatrix} = \begin{vmatrix} 25 \\ -5 \end{vmatrix}; \quad K_u = \begin{vmatrix} -5 & 25 \\ 1 & -5 \end{vmatrix}; \; \Delta_1 = -5 \neq 0; \; \Delta_2 = 0; \; rank\, K_u = 1,$$

i.e. $rank\, K_u \neq n$

Hence, the researched system is not controllable by R. Kallman's criterion because the rank of the block matrix of controllability is not equal to order of system.

II. Check on controllability by E. Gilbert's criterion

Algorithm and solution

1. The dynamic system is described in state-space by the system of the equations (*).

Through linear transformation of coordinates $X = V \cdot X^*$, where V is a modal matrix of an initial matrix A, we will transform a system (*) to a diagonal form of the following look:

$$X = VX^*$$
$$V\dot{X}^* = AVX^* + BU$$
$$V^{-1}V\dot{X}^* = V^{-1}AVX^* + V^{-1}BU$$

(**)

$$\begin{cases} \dot{X}^* = \Lambda X^* + B^* U \\ Y^* = C^* X^* \end{cases}.$$

There are $\Lambda = V^{-1}AV$, $B^* = V^{-1}B$, $C^* = CV$.

The matrix Λ is scalar matrix which have own numbers of a matrix of *A* on diagonal. Hence, a scalar matrix Λ is equal:

$$\Lambda = \begin{vmatrix} -5 & 0 \\ 0 & 1 \end{vmatrix}.$$

2. We define own matrixes of a vector of V_i $\forall$ $i=1,n$ from the following identity:

$$\lambda_i V_i = AV_i,,$$

$$V = |V_1 V_2| = \begin{vmatrix} v_{11} & v_{21} \\ v_{12} & v_{22} \end{vmatrix}.$$

for i=1:

$$-5\begin{vmatrix} v_{11} \\ v_{12} \end{vmatrix} = \begin{vmatrix} -4 & 5 \\ 1 & 0 \end{vmatrix}\begin{vmatrix} v_{11} \\ v_{12} \end{vmatrix}.$$

We will write in a scalar form:

$$\begin{cases} -5v_{11} = -4v_{11} + 5v_{12} \\ -5v_{12} = v_{11} \end{cases}; \text{ let } v_{12}=1, \text{ then } v_{11}= -5.$$

Hence,

$$V_1 = \begin{vmatrix} -5 \\ 1 \end{vmatrix};$$

for i=2 write down: $\begin{vmatrix} v_{21} \\ v_{22} \end{vmatrix} = \begin{vmatrix} -4 & 5 \\ 1 & 0 \end{vmatrix} \cdot \begin{vmatrix} v_{21} \\ v_{22} \end{vmatrix}$; $\begin{cases} v_{21} = -4v_{21} + 5v_{22} \\ v_{22} = v_{21} \end{cases}$;

let $v_{21}=1$, then $v_{22}= 1$.

hence,

$$V_2 = \begin{vmatrix} 1 \\ 1 \end{vmatrix}.$$

We write down of the matrix of own vectors:

$V = \begin{vmatrix} -5 & 1 \\ 1 & 1 \end{vmatrix}$; determinant *det V= -6 ≠0 is not equal to zero, therefore, there is an inverse matrix* V^{-1}.

3. We define of inverse matrix V^{-1}

Finding of an inverse matrix is carried out in three stages:

a) We create a matrix of algebraic complements of *M* which elements we receive elimination of a line and column and multiplication by $(-1)^{(i+j)}$, where 'i' is a number of the eliminated line, 'j' is a number of the eliminated column:

$$M = \begin{vmatrix} 1 & -1 \\ -1 & 5 \end{vmatrix}.$$

b) We write a matrix of *D* which elements are elements of the transposed matrix M^T:

$$D = \begin{vmatrix} 1 & -1 \\ -1 & -5 \end{vmatrix}.$$

c) We define of inverse matrix V^{-1}:

$$V^{-1} = \frac{D}{\det V}. \qquad \text{Here is } V^{-1} = -1/6 \begin{vmatrix} 1 & -1 \\ -1 & -5 \end{vmatrix}.$$

Check: $V^{-1} V = I_n$? $\quad -1/6 \begin{vmatrix} 1 & -1 \\ -1 & -5 \end{vmatrix} \begin{vmatrix} -5 & 1 \\ 1 & 1 \end{vmatrix} = -1/6 \begin{vmatrix} -6 & 0 \\ 0 & -6 \end{vmatrix} = \begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix} = I_2.$

4. We carry out check of controllability by E. Gilbert's criterion:

$$\Lambda \overset{\Delta}{=} diag\ (\lambda_1, \lambda_2, \ldots, \lambda_n), \quad \text{i.e.} \quad \Lambda = \begin{vmatrix} -5 & 0 \\ 0 & 1 \end{vmatrix}.$$

Obtain B^*: $\qquad B^* = V^{-1} B; \quad B^* = -1/6 \begin{vmatrix} 1 & -1 \\ -1 & -5 \end{vmatrix} \begin{vmatrix} -5 \\ 1 \end{vmatrix} = \begin{vmatrix} 1 \\ 0 \end{vmatrix};$

Hence,

$$\dot{X}^* = \begin{vmatrix} -5 & 0 \\ 0 & 1 \end{vmatrix} X^* + \begin{vmatrix} 1 \\ 0 \end{vmatrix} u \ \text{; we write down in scalar form} \ \begin{cases} \dot{x}_1^* = -5x_1^* + u_1 \\ \dot{x}_2^* = x_2^* \end{cases}.$$

Hence, the researched system is not controllable by R. Gilbert's criterion because matrix B^* contains a zero line that is visually visible in a scalar form as $u_2 = 0$.

General conclusion: The moving of the researched system is unstable across Lyapunov and this system is uncontrollable by R. Kallman and E. Gilbert's criteria.

Task Investigate a dynamic system on controllability by R. Kallman and E. Gilbert's criteria if the mathematical description of a system is given in the state-space in the following look:

$$\begin{cases} \dot{x} = Ax + Bu \\ y = Cx \end{cases},$$

where matrix *A*, *B*, *C* are matrixes with constant coefficients (on variants).

Variants:

1)

$$A = \begin{vmatrix} 2 & 3 \\ -1 & 6 \end{vmatrix}, \; B = \begin{vmatrix} 1 \\ -2 \end{vmatrix}, \; C = \begin{vmatrix} -3 \\ 2 \end{vmatrix}.$$

2)

$$A = \begin{vmatrix} -8 & 15 \\ 2 & -7 \end{vmatrix}, \; B = \begin{vmatrix} 5 \\ 6 \end{vmatrix}, \; C = \begin{vmatrix} -1 \\ -2 \end{vmatrix}.$$

3)

$$A = \begin{vmatrix} 7 & 2 \\ 4 & 5 \end{vmatrix}, \; B = \begin{vmatrix} 2 \\ 1 \end{vmatrix}, \; C = \begin{vmatrix} 0 \\ 1 \end{vmatrix}.$$

4)

$$A = \begin{vmatrix} 7 & 9 \\ 6 & 4 \end{vmatrix}, \; B = \begin{vmatrix} -1 \\ 0 \end{vmatrix}, \; C = \begin{vmatrix} 2 \\ 0 \end{vmatrix}.$$

5)

$$A = \begin{vmatrix} 5 & 6 \\ 8 & 7 \end{vmatrix}, \; B = \begin{vmatrix} 1 \\ 3 \end{vmatrix}, \; C = \begin{vmatrix} 2 \\ -1 \end{vmatrix}.$$

6)

$$A = \begin{vmatrix} 1 & -1 \\ 7 & 9 \end{vmatrix}, \; B = \begin{vmatrix} -3 \\ 0 \end{vmatrix}, \; C = \begin{vmatrix} 1 \\ -3 \end{vmatrix}.$$

7)

$$A = \begin{vmatrix} 2 & 6 \\ 8 & 4 \end{vmatrix}, \; B = \begin{vmatrix} 5 \\ -2 \end{vmatrix}, \; C = \begin{vmatrix} -1 \\ 2 \end{vmatrix}.$$

8)

$$A = \begin{vmatrix} -5 & 4 \\ -2 & -2 \end{vmatrix}, \; B = \begin{vmatrix} 7 \\ 1 \end{vmatrix}, \; C = \begin{vmatrix} 3 \\ -2 \end{vmatrix}.$$

9)

$$A = \begin{vmatrix} -8 & -4 \\ -2 & -6 \end{vmatrix}, \; B = \begin{vmatrix} -2 \\ 9 \end{vmatrix}, \; C = \begin{vmatrix} 0 \\ -2 \end{vmatrix}.$$

10)

$$A = \begin{vmatrix} 9 & 9 \\ 2 & 6 \end{vmatrix}, \ B = \begin{vmatrix} -3 \\ 0 \end{vmatrix}, \ C = \begin{vmatrix} 1 \\ -3 \end{vmatrix}.$$

11)

$$A = \begin{vmatrix} 2 & -1 \\ -1 & 2 \end{vmatrix}, \ B = \begin{vmatrix} 4 \\ -3 \end{vmatrix}, \ C = \begin{vmatrix} -1 \\ 3 \end{vmatrix}.$$

12)

$$A = \begin{vmatrix} 10 & 11 \\ 14 & 13 \end{vmatrix}, \ B = \begin{vmatrix} -1 \\ 1 \end{vmatrix}, \ C = \begin{vmatrix} 1 \\ 2 \end{vmatrix}.$$